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Unit A1.4, Describing Functions: Notes Name ______________________________

Lesson 1: What Is a Function?

Summary

A *function* is a rule that assigns exactly one output to each possible input.

When determining if a rule is a function, a table can be used to organize inputs and outputs. If one input has multiple possible outputs, then the rule is not a function.

Rule H takes any measurement in meters and converts it to centimeters.

Input	Output
3 m	300 cm
2.6 m	260 cm
5.5 m	550 cm
3 m	300 cm

In this relationship, each input has exactly one output, so it's a function.

For instance, when inputting 3 m, the output will always be 300 cm.

Rule J takes whole numbers from 1 to 15 and outputs a word of that length.

Input	Output
5	watch
9	vegetable
9	classroom
1	a

In this relationship, inputs have multiple possible outputs, so it's not a function.

For instance, the table shows that the input 9 has two different outputs ("vegetable" and "classroom").

Rule K takes any year and returns the last two digits.

Input	Output
2009	09
1915	15
2015	15
2012	12

Decide if **Rule K** is a function. Explain your thinking.

Things I Want to Remember

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Unit A1.4, Describing Functions: Notes Name ______________________________

Lesson 1: What Is a Function?

Try This!

Rules L and M are functions. Complete the remaining inputs and outputs.

1.1 **Rule L** takes a value and outputs that value plus one.

Input	Output
-5	-4
6	7
2	
-5	

1.2 **Rule M** takes a value and outputs that value multiplied by 10.

Input	Output
2	20
60	600
0.8	
	70

2. Circle the rule that is **not** a function.

Rule P takes any word and outputs the number of letters.

Input	Output
at	2
tree	4
simple	6
the	3

Rule Q takes any month and outputs its calendar order.

Input	Output
January	1
March	3
April	4
March	3

Rule R takes any value and either multiplies or divides it by 2.

Input	Output
2	4
10	20
3	6
2	1

Rule S takes a letter and shifts it one place forward in the alphabet.

Input	Output
C	D
M	N
Z	A
O	P

- ☐ I can decide whether or not a rule is a function.
- ☐ I can explain that a function has only one output for every input.

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Unit A1.4, Describing Functions: Notes Name ________________________________

Lessons 2–3: Function Notation and Equations

Summary

Function notation is a way of writing the inputs and outputs of a function.

For example, suppose we made a function for determining the price of a medium pizza.

The table shows some inputs and outputs.

MENU
Small: $13.50 plus $1.25 per topping
Medium: $15.50 plus $1.50 per topping
Large: $17.75 plus $2.25 per topping

Number of Toppings	Price
0	$\$15.50$
1	$\$17.00$
2	$\$18.50$

$m(2)$ is an example of a statement in function notation and is pronounced "m of two."

$m(2) = 18.50$ means:

The price of a medium pizza with 2 toppings is $\$18.50$.

$m(1) = 17.00$ means:

____

We can define the function $m(t)$ using an equation.

$m(t) = 15.5 + 1.5t$ represents the cost of a medium pizza with t toppings.

What is the value of $m(4)$?

$m(4) = 15.5 + 1.5(4)$

$m(4) = 21.5$

$s(t) =$ ______________________ represents the cost of a small pizza with t toppings.

What is the value of $s(2)$?

Things I Want to Remember

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Unit A1.4, Describing Functions: Notes Name ______________________________

Lessons 2–3: Function Notation and Equations

Try This!

An ice cream shop serves ice cream in either a waffle cone or a bowl. Customers also decide how many scoops of ice cream they want.

$w(x) = 1.25x + 2.5$ represents the cost of a waffle cone order, where x represents the number of scoops of ice cream.

1.1 What is the value of $w(2)$?

1.2 What does $w(4) = 7.5$ mean in the context of the ice cream shop?

1.3 $b(x)$ represents the cost of a bowl order, where x represents the number of scoops of ice cream. The bowl costs $\$1$ plus $\$1.50$ for each scoop of ice cream. Write an equation for $b(x)$.

$b(x) =$

1.4 Fatima compares her ice cream order to her brother's order. What does the statement $w(2) > b(4)$ mean?

- ☐ I can interpret a statement that uses function notation in context.
- ☐ I can evaluate functions written in function notation.
- ☐ I can write equations of functions using function notation.

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Unit A1.4, Describing Functions: Notes Name ______________________

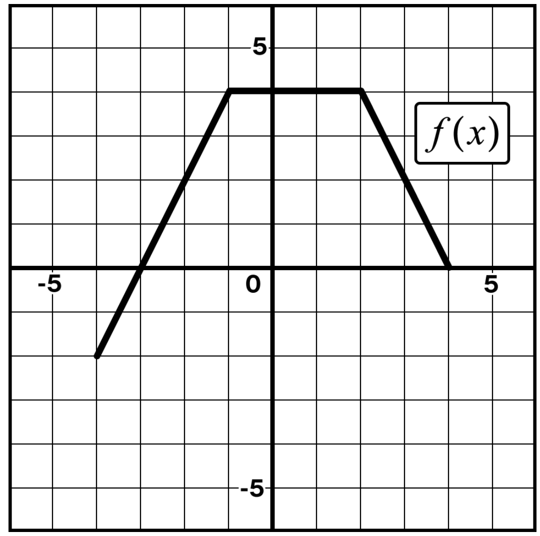
Lessons 4: Function Notation and Graphs

Summary

Function notation, like $f(x)$, describes the name of the function, the input, and the output. The input, x, and the output, $f(x)$, can be represented as the coordinate pair $(x, f(x))$.

Complete the table using the graph of $f(x)$.

Function Notation Statement	Coordinate Pair
$f(-4) = -2$	$(-4, -2)$
$f(0) = 4$	$(0, 4)$
$f(4) = 0$	
	$(3, 2)$



Function statement comparisons can be made using the graph as well.

Function Notation Statement	True or False?	Explain
$f(-1) > 3$	True	$f(-1) = 4$ $4 > 3$ is true
$f(-4) = f(4)$	False	$f(-4) = -2$ $f(4) = 0$ $-2 = 0$ is false
$f(-1) = f(1)$		
$f(-2) > f(4)$		

Things I Want to Remember

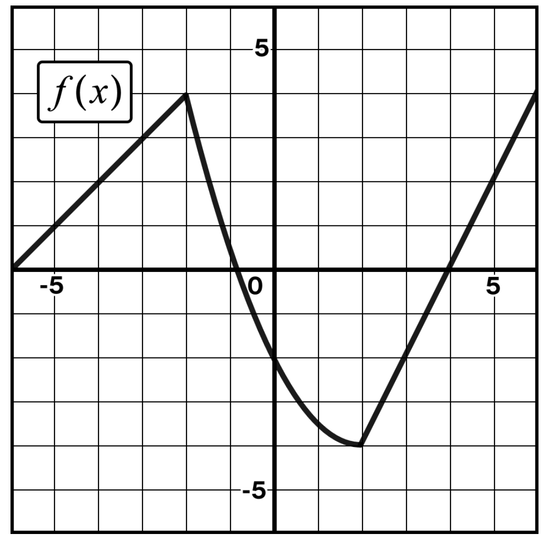
Unit A1.4, Describing Functions: Notes Name ________________________________

Lessons 4: Function Notation and Graphs

Try This!

1.1 Evaluate the function notation statements.

- $f(-4) =$
- $f(-2) =$
- $f(0) =$
- $f(5) =$



1.2 Select **all** the true statements.

- ☐ $f(-2) = f(5)$
- ☐ $f(-4) = f(5)$
- ☐ $f(5) > f(0)$
- ☐ $f(-4) < f(0)$
- ☐ $f(-2) > f(5)$

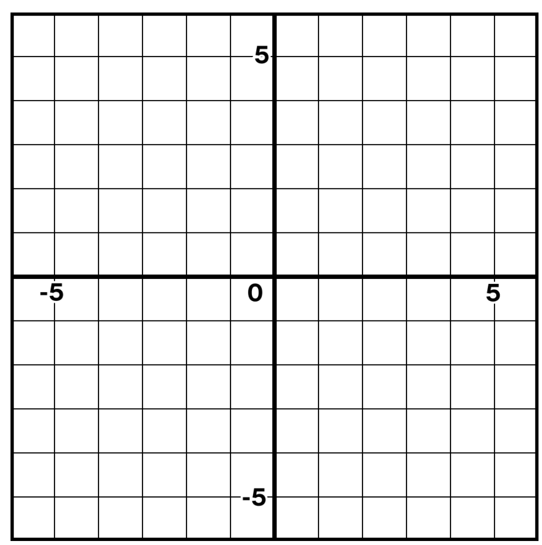
Graph the coordinate pair that corresponds to each function statement.

2.1 $g(-3) = 1$

2.2 $g(0) = 3$

2.3 $g(2) = -2$

2.4 $g(4) = -4$



☐ I can connect statements written in function notation to the function's graph.

Unit A1.4, Describing Functions: Notes Name ______________________________

Lessons 5–6: Key Features of Graphs

Summary

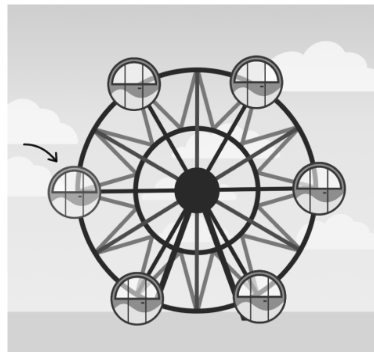
A graph can reveal in more detail what is happening to the inputs and outputs during a situation.

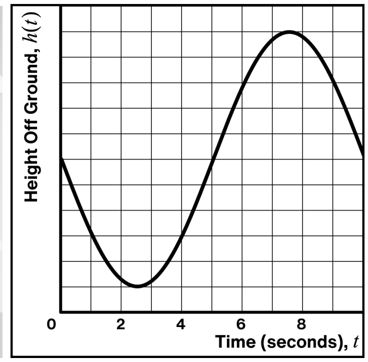
Here $h(t)$ represents the height of the cart on the Ferris wheel at time t.

When is the Ferris wheel at its lowest height?

Around 2.5 *seconds*

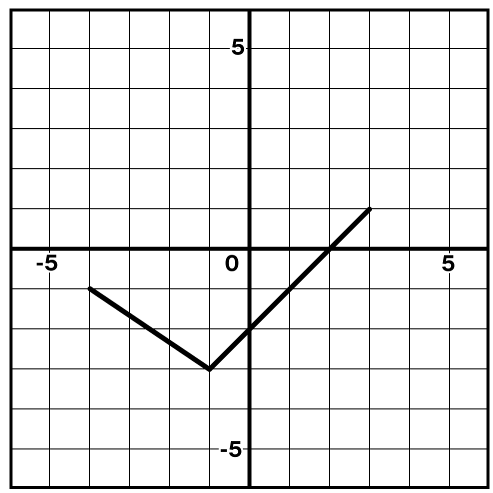
Which is greater: $h(1)$ or $h(3)$?





The terms *maximum*, *minimum*, *positive*, *negative*, *increasing*, and *decreasing* can be used to describe parts of a graph. Complete the table.

Key Features	When
Minimum: Coordinates of the lowest point of the graph	$(-1, -3)$
Maximum: Coordinates of the highest point of the graph	
Positive: When the function has positive outputs. The graph is above the x-axis.	$x > 2$
Negative:	$x < 2$
Increasing: When the function's outputs increase as the inputs increase; graph is upward-sloping, left to right	$x > -1$
Decreasing: When the function's outputs decrease as the inputs increase; graph is downward-sloping, left to right	



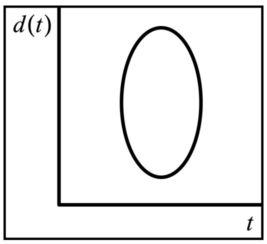
Things I Want to Remember

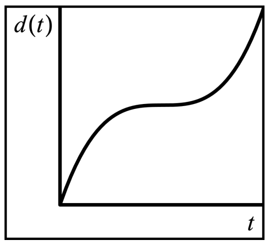
Lessons 5–6: Key Features of Graphs

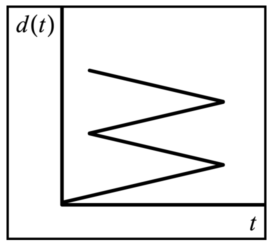
Try This!

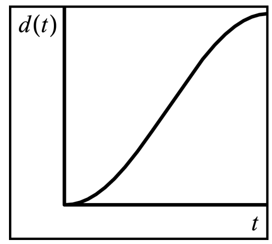
1. Jasmine races around an oval track. $d(t)$ represents how many meters were run at time t. Select all possible graphs that could represent Jasmine's race.

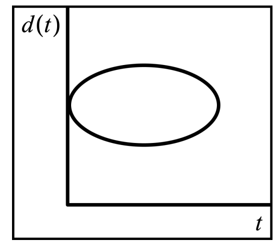
☐ Graph 1 ☐ Graph 2 ☐ Graph 3 ☐ Graph 4 ☐ Graph 5



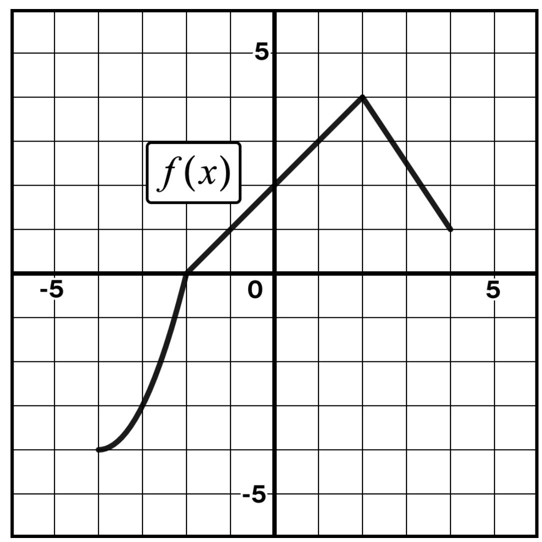








Circle **all** descriptions that apply to the specified interval of $f(x)$.



2.1 $x < -2$

positive negative increasing decreasing

2.2 $x > 2$

positive negative increasing decreasing

3.1 What is the maximum? __________

3.2 What is the minimum? __________

☐ I can sketch a graph of a function to match a situation.
☐ I can make connections between situations and graphs.
☐ I can describe the key features of a graph using words like *positive*, *negative*, *maximum*, *minimum*, *increasing*, and *decreasing*.
☐ I can use the key features of a function to build a graph of a function.

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Unit A1.4, Describing Functions: Notes Name ______________________________

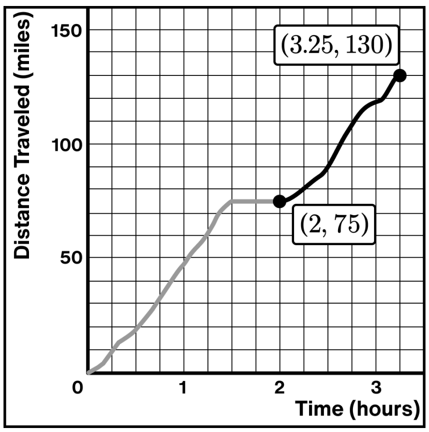
Lesson 7: Average Rate of Change

Summary

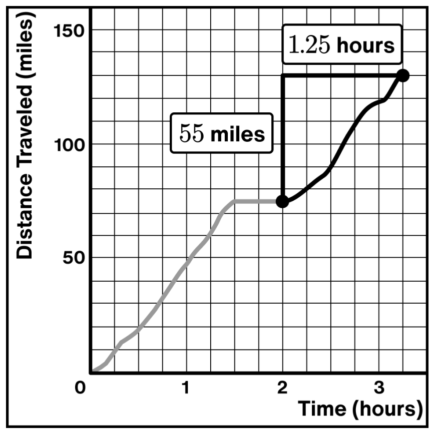
The *average rate of change* is equivalent to the slope of the line between two points.

Let's look at an interval of Arjun's car trip below. To determine the average rate of change between 2 to 3.25 hours, divide the change in distance (55 miles) by the change in time (1.25 hours). Two approaches for doing that are shown below.

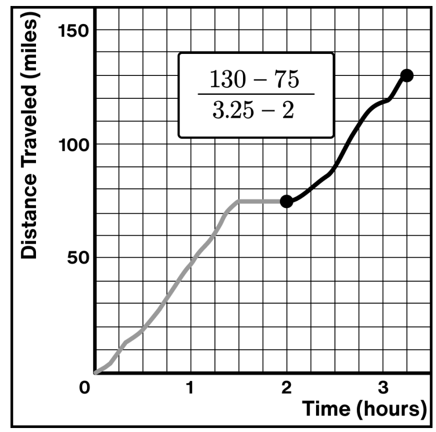
Arjun's Car Trip



Approach 1

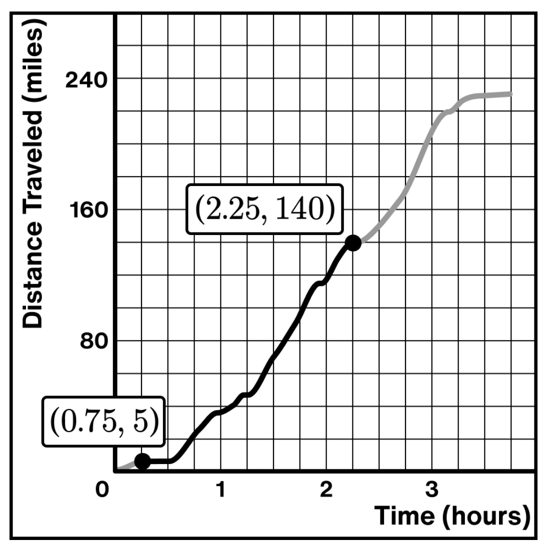


Approach 2



The average rate of change for the interval 2 to 3.25 is 44. That means Arjun's average speed was 44 miles per hour in that interval.

Here is Troy's trip. Determine Troy's average rate of change from 0.25 to 2.25 hours.

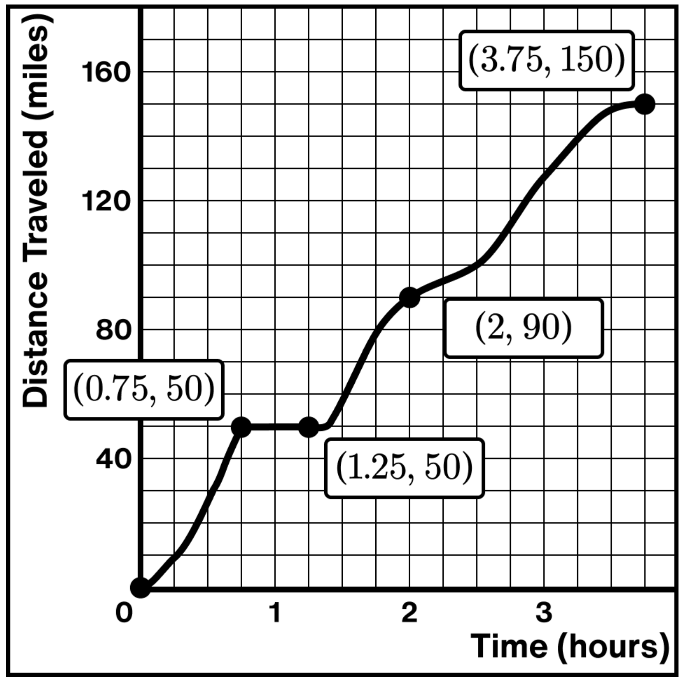


Things I Want to Remember

Lesson 7: Average Rate of Change

Try This!

Oscar took the train to attend his friend's birthday. This graph represents his trip.



1.1 Which interval had the greater average rate of change?

0 to 0.75 hours 0 to 1.25 hours

1.2 Calculate the average rate of change for each interval.

Interval	**Average Rate of Change (mph)**
0 to 3.75 hours	
0.75 to 2 hours	
0.75 to 1.25 hours	

1.3 What could have happened during the interval 0.75 to 1.25 hours?

A. The train was traveling at a constant speed of 50 miles per hour.

B. The train was traveling on a straight track during that time.

C. The train stopped completely to wait for the track to clear.

D. The train traveled east for 30 minutes.

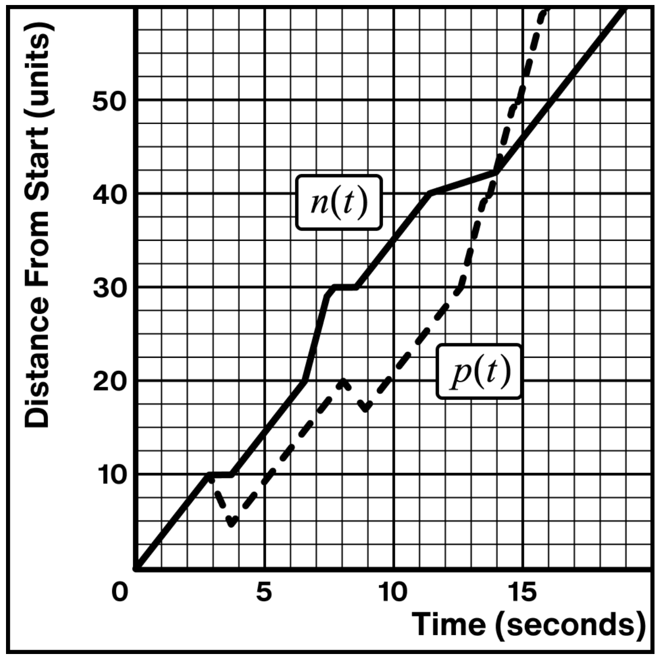
☐ I can calculate the average rate of change over an interval on a graph.
☐ I can interpret the average rate of change in context.

Lesson 8: Comparing Graphs

Summary

When analyzing two or more functions, you can compare the key features and behavior of different parts of their graphs.

Nekeisha and Polina raced their spaceships. Functions $n(t)$ and $p(t)$ give their spaceships' distance after t seconds.



Statement	Meaning
$n(6) > p(6)$	At 6 seconds, Nekeisha was ahead of Polina.
$n(16) < p(16)$	
$n(3) = p(3)$	At 3 seconds, Nekeisha and Polina both traveled the same distance.
	At 14 seconds, Nekeisha and Polina both traveled the same distance.

Decide if each statement is true, false, or cannot be determined.

$n(t)$ and $p(t)$ are both decreasing from 8 to 9 seconds.

True False Cannot be determined

$n(t)$ and $p(t)$ have the same average rate of change from 0 to 14 seconds.

True False Cannot be determined

Things I Want to Remember

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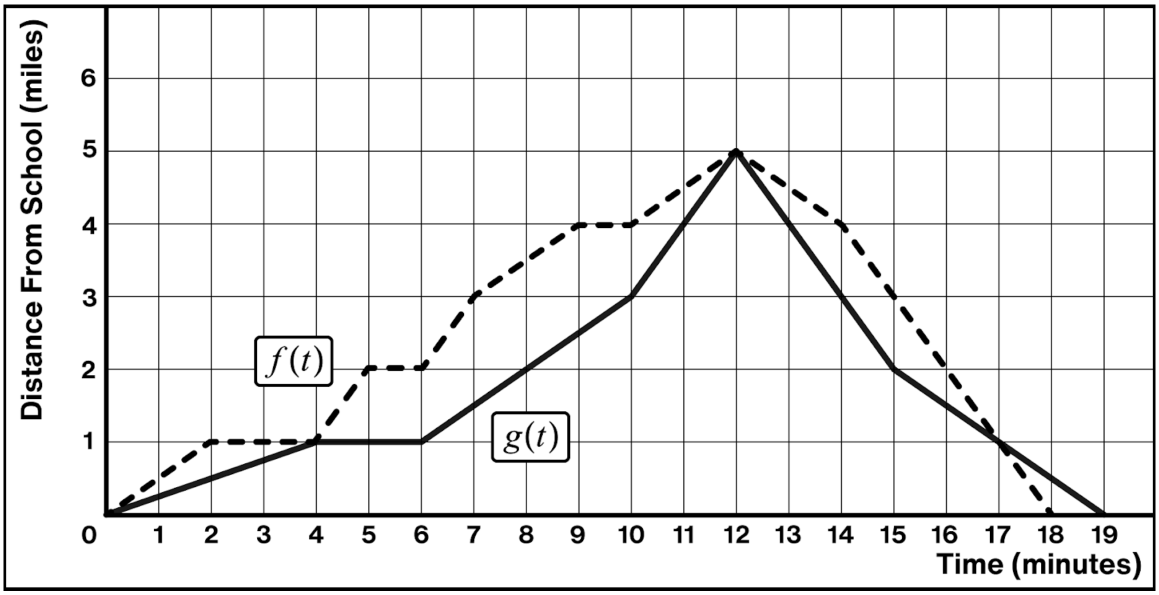
Unit A1.4, Describing Functions: Notes Name ______________________________

Lesson 8: Comparing Graphs

Try This!

A school has two buses that take different routes to drop students off. They leave at the same time. $f(t)$ and $g(t)$ represent the distance of each bus from school (in miles) after t minutes.



1.1 Select **all** the true statements.

- ☐ $f(2) = g(2)$
- ☐ $f(15) > g(15)$
- ☐ $f(10) = g(10)$
- ☐ $f(2) = 6$
- ☐ $f(8) > g(8)$

1.2 Write one value of t where $f(t) = g(t)$.

1.3 Select **all** the true statements.

- ☐ $f(t)$ and $g(t)$ have the same maximum.
- ☐ $f(t)$ and $g(t)$ are both increasing from 4 to 5 minutes.
- ☐ $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes.
- ☐ $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes.
- ☐ $f(t)$ and $g(t)$ have the same average rate of change from 6 to 12 minutes.

☐ I can compare two graphs of functions using their key features.
☐ I can use function notation to compare two graphs of functions.

Unit A1.4, Describing Functions: Notes Name ______________________________

Lesson 9: Introducing Domain and Range

Summary

The *domain* of a function is the set of all possible input values. The domain can be described in words and in symbols.

When determining the domain of a situation or function, consider if a variety of inputs are possible, such as negative numbers, positive numbers, fractions, large values, and zero.

Let's look at two different frozen yogurt shops' pricing models.

$f(w) = 3 + 0.5w$ represents the cost of a frozen yogurt that weighs w ounces.

Which inputs make sense?

-2	0	0.5	2	20

Describe the domain.

All numbers greater than 0. It is not possible to have negative ounces and it is not possible to make an order of 0 ounces.

$g(t) = 5 + 0.25t$ represents the cost of a frozen yogurt with t toppings (up to 4 toppings).

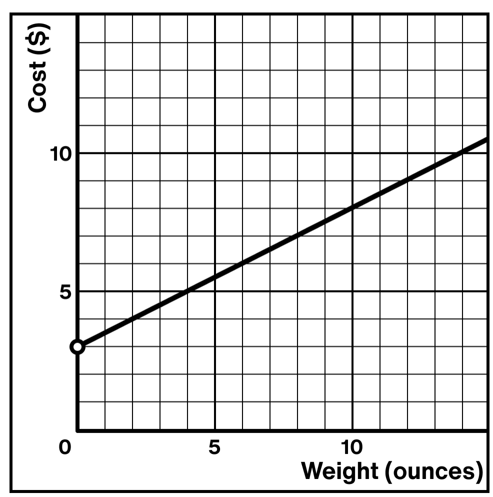
Which inputs make sense?

-2	0	0.5	2	20

Describe the domain.

The *range* of a function is the set of all possible output values. Similar to the domain, consider a variety of outputs and use the situation or graph to help you make sense of the possibilities.

The function $f(w) = 3 + 0.5w$ is graphed.



What are possible outputs that make sense?

Describe the range.

Things I Want to Remember

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Unit A1.4, Describing Functions: Notes Name ______________________

Lesson 9: Introducing Domain and Range

Try This!

Match each situation with a domain description.

1.1 _____

A delivery company charges a $\$5.00$ fee and $\$1.25$ for every pound a package weighs. $d(x) = 5 + 1.25x$ represents the cost of delivery for a package that weighs x pounds.

1.2 _____

A boba shop sells milk tea for $\$5.00$ and charges $\$1.25$ for every add-on (up to 5 add-ons). $b(x) = 5 + 1.25x$ represents the cost of a milk tea with x add-ons.

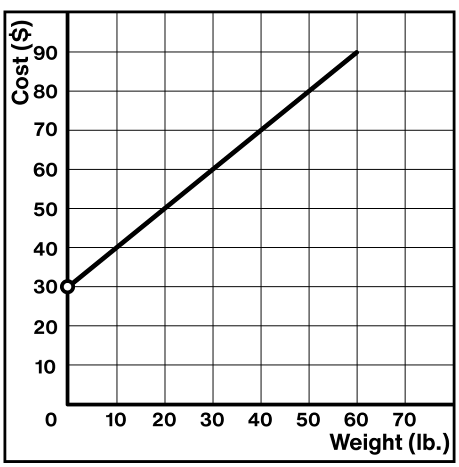
1.3 _____

A student graphs $f(x) = 5 + 1.25x$.

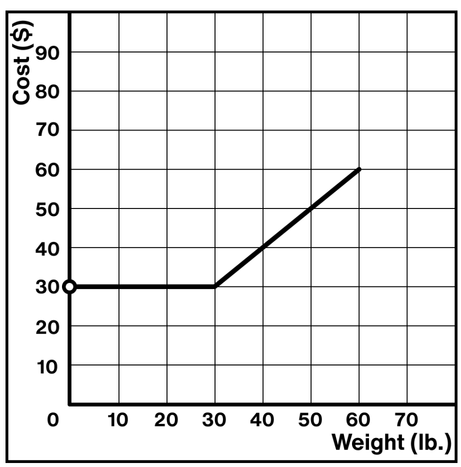
A. All numbers	**B.** All numbers greater than 0	**C.** Whole numbers from 0 to 5

Here are three graphs of different airlines' luggage costs as a function of weight.
Match each graph with a range description.

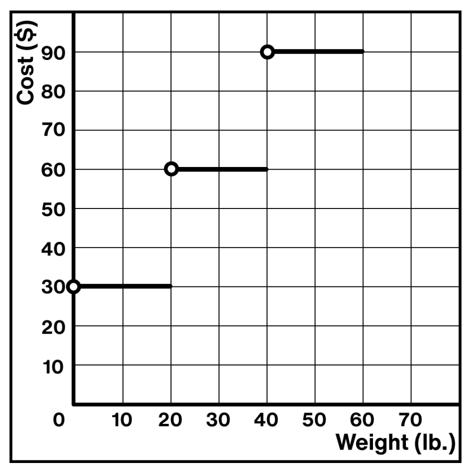
2.1 _____



2.2 _____



2.3 _____



A. All numbers from 30 to 60	**B.** $30, 60, 90$	**C.** Numbers greater than 30 and less than or equal to 90

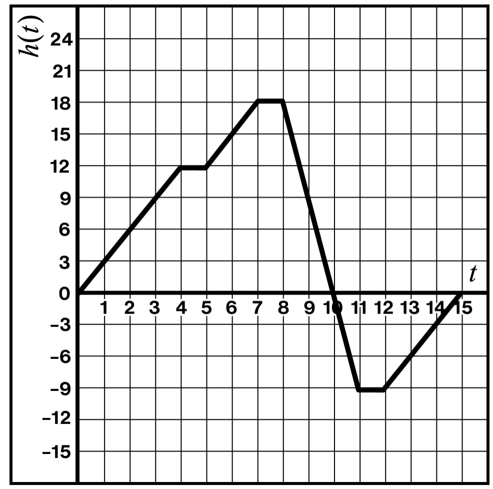
- ☐ I can decide if a value is a possible input or output for a function in context.
- ☐ I can describe the domain and range of a function as the set of all possible inputs and outputs.

Lessons 10–11: Domain and Range of Graphs

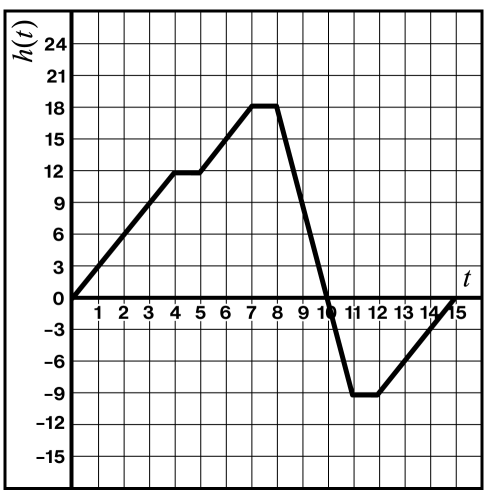
Summary

The domain and range of functions can be described using a *compound inequality*, which is two or more inequalities joined together.

Let's look at a guest's elevator ride at the Four Quadrants Hotel. The graph shows $h(t)$, the height of the elevator in meters, t seconds into the guest's ride.



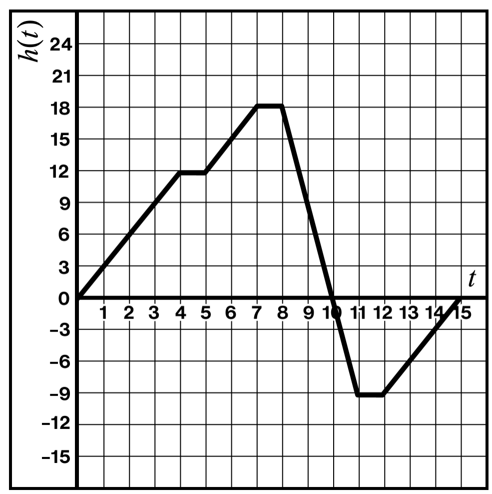
Sketch where you see the domain of $h(t)$.



Complete the compound inequality to describe the domain of $h(t)$.

$\_\_\_\_\_ \leq t \leq \_\_\_\_\_$

Sketch where you see the range of $h(t)$.



Complete the compound inequality to describe the range of $h(t)$.

$\_\_\_\_\_ \leq h(t) \leq \_\_\_\_\_$

Things I Want to Remember

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Unit A1.4, Describing Functions: Notes Name ______________________

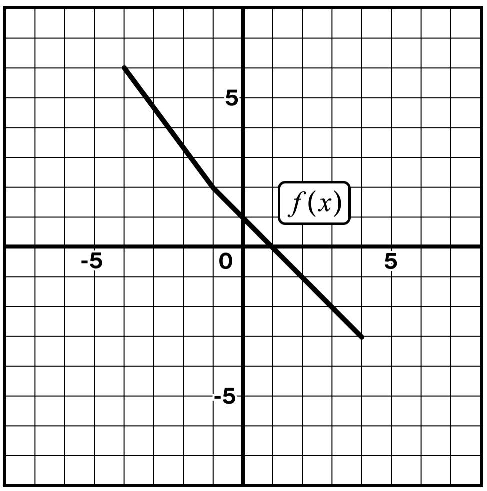
Lessons 10–11: Domain and Range of Graphs

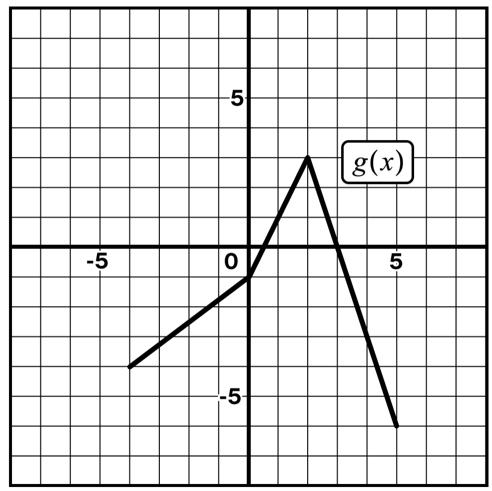
Try This!

Complete the compound inequalities to describe the domain and range of each function.

1.1 Domain	1.2 Range
___ $\leq x \leq$ ___	___ $\leq f(x) \leq$ ___

2.1 Domain	2.2 Range
___ $\leq x \leq$ ___	___ $\leq g(x) \leq$ ___



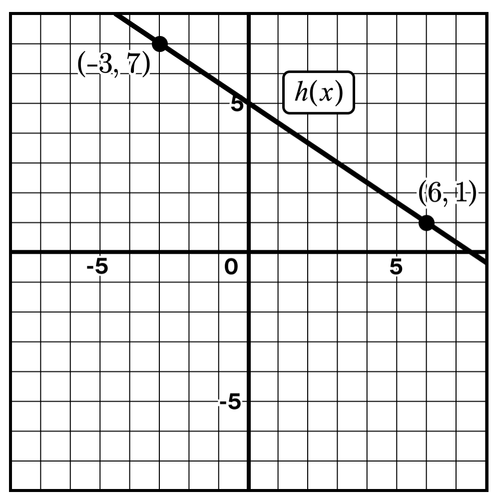


3.1 Write a domain that could restrict the graph of $h(x)$ from $(-3, 7)$ to $(6, 1)$.

___ $\leq x \leq$ ___

3.2 Write a range that could restrict the graph of $h(x)$ from $(-3, 7)$ to $(6, 1)$.

___ $\leq h(x) \leq$ ___



- ☐ I can write the domain and range of a function using inequalities.
- ☐ I can interpret the meaning of the domain and range in context.
- ☐ I can restrict the domain and range of a function using inequalities.

Lesson 12: Functions in Context

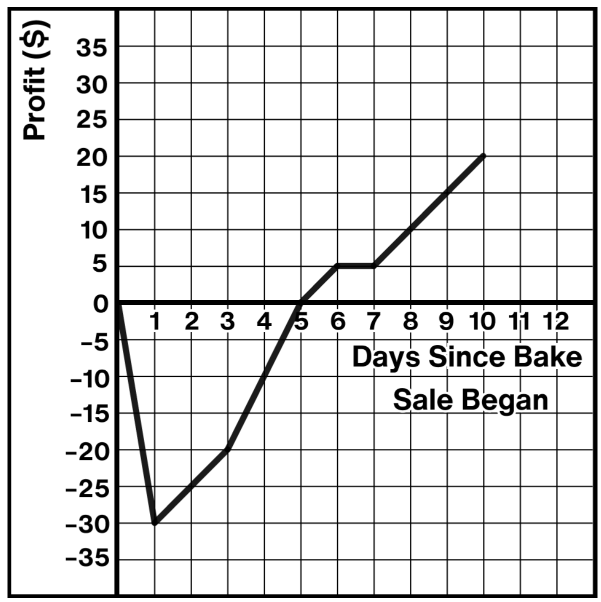
Summary

A function's graph can be described using key features, which can be interpreted when provided a context.

Let's look at Kayleen's bake sale experience at her school. Kayleen decided to make cakes for her school's bake sale and tracked her profits.

Complete the table with interpretations of each term in this context.

Term	Meaning
maximum	
negative interval	Kayleen has not made her money back.
positive interval	
decreasing interval	Kayleen spends money on materials to make cakes.
increasing interval	



Tell a story about Kayleen's bake sale experience that makes sense based on the graph.

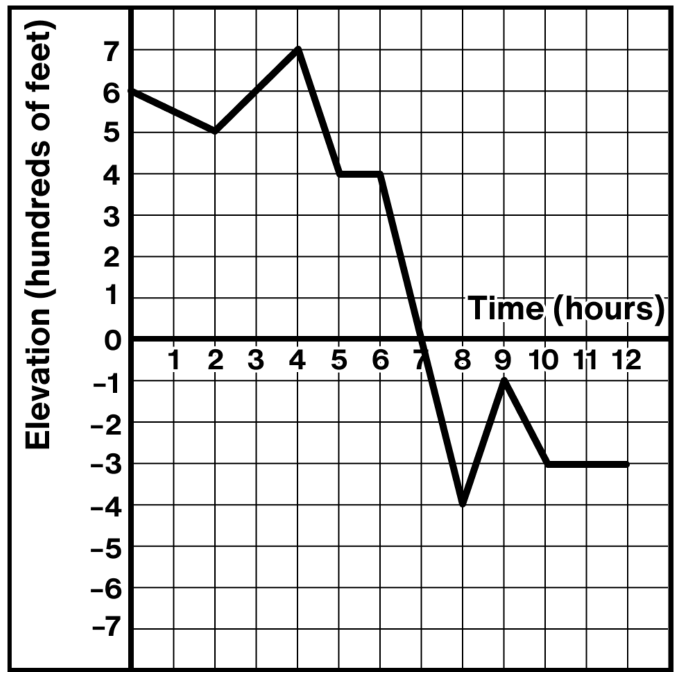
Things I Want to Remember

 Name ______________________

Lesson 12: Functions in Context

Try This!

Parv hiked down to the bottom of a canyon and tracked his elevation above and below sea level. The graph shows $p(t)$, Parv's elevation after t hours.



1.1 Calculate the average rate of change of Parv's hike from 0 to 12 hours.

1.2 Complete the table.

Term	Meaning
minimum	
increasing interval	
decreasing interval	
domain	
range	

☐ I can interpret the key features of a function in context.

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Unit A1.4, Describing Functions: Notes Name ______________________________

Lessons 13–14: Piecewise-Defined Functions

Summary

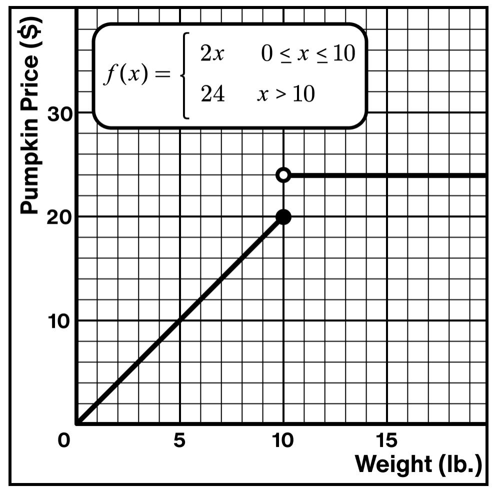
A *piecewise-defined function* is a function in which different rules apply to different intervals in its domain.

At Omar's Farm, the function $f(x)$ represents the price of a pumpkin with a weight of x pounds. Pumpkins 10 pounds or less cost \$2 per pound, and pumpkins more than 10 pounds cost \$24.

When $0 \le x \le 10$, $f(x) = 2x$.

When $x > 10$, $f(x) = 24$.

Evaluate	Interval	Equation	Calculate
$f(4)$	4 is in $0 \le x \le 10$	$f(x) = 2x$	$f(4) = 2(4)$ $= 8$
$f(15)$	15 is in $x > 10$	$f(x) = 24$	$f(15) = 24$
$f(10)$			
$f(11)$			

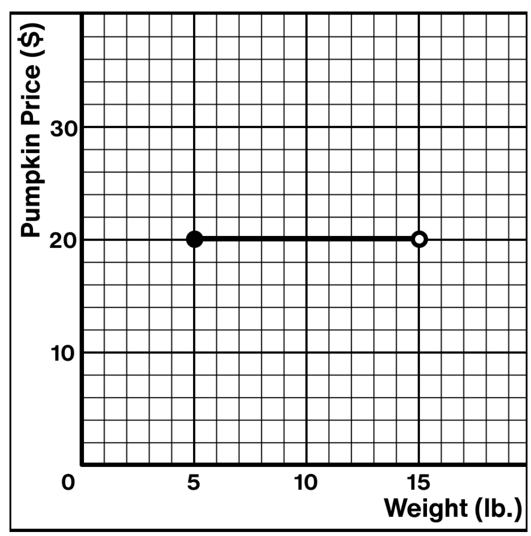


The farm is changing their prices to the following:

- Pumpkins less than 5 pounds: \$10
- Pumpkins greater than or equal to 15 pounds: \$30
- All other pumpkins: \$20

Complete the piecewise-defined function and the graph.

$$f(x) = \begin{cases} 10 & \boxed{\phantom{5 \le x < 15}} \\ \boxed{} & 5 \le x < 15 \\ 30 & \boxed{\phantom{5 \le x < 15}} \end{cases}$$



Things I Want to Remember

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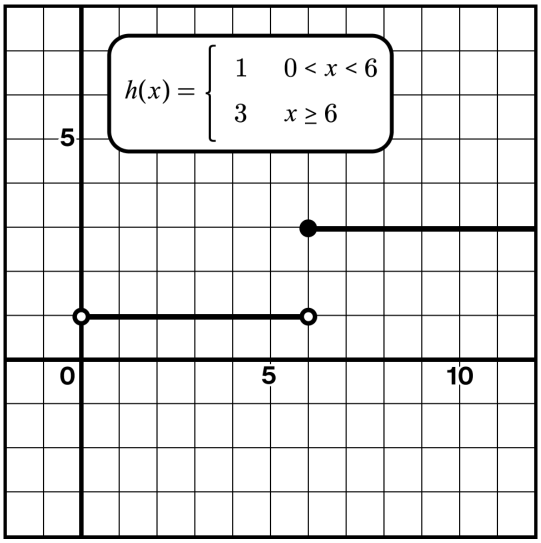
Unit A1.4, Describing Functions: Notes Name ______________________

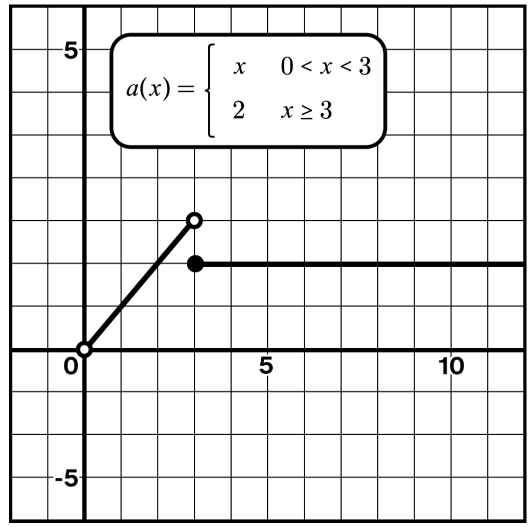
Lessons 13–14: Piecewise-Defined Functions

Try This!



1.1 What is $h(4)$?

1.2 What is $h(6)$?



2.1 What is $a(1)$?

2.2 What is $a(10)$?

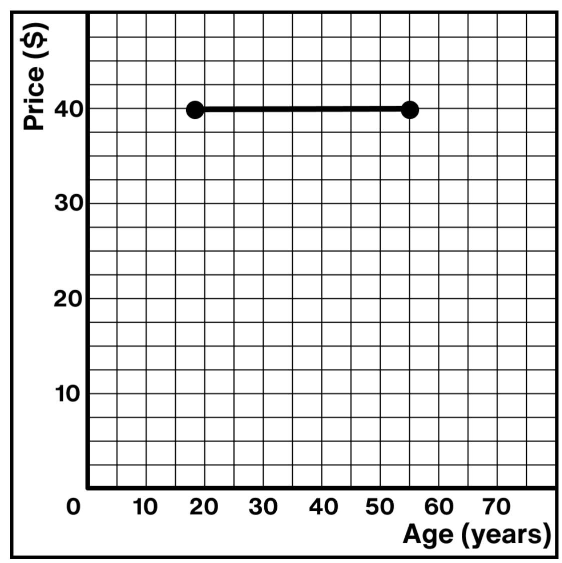
The Desmopolis Transportation Agency is considering using a passenger's age to determine the price of a train ticket. The function $p(x)$ gives the price of a ticket for a person who is x years old.

One plan suggested the following ticket prices.

- Younger than 20 years old: $\$30$
- Older than 55 years old: $\$20$
- All other ages: $\$40$

3.1 Complete the piecewise-defined function and the graph.

$$p(x) = \begin{cases} 30 & \boxed{} \\ \boxed{} & 20 \leq x \leq 55 \\ 20 & \boxed{} \end{cases}$$



- ☐ I can read and understand a piecewise-defined function.
- ☐ I can explain how a piecewise-defined function represents a situation.
- ☐ I can evaluate a piecewise-defined function in function notation.
- ☐ I can use information from a situation to write equations of piecewise-defined functions.
- ☐ I can sketch a graph of a piecewise-defined function.

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Unit A1.4, Describing Functions: Notes Name ________________________________

Lessons 15–16: Absolute Value Functions

Summary

The output of an *absolute value function* is the distance of its input from a given value.

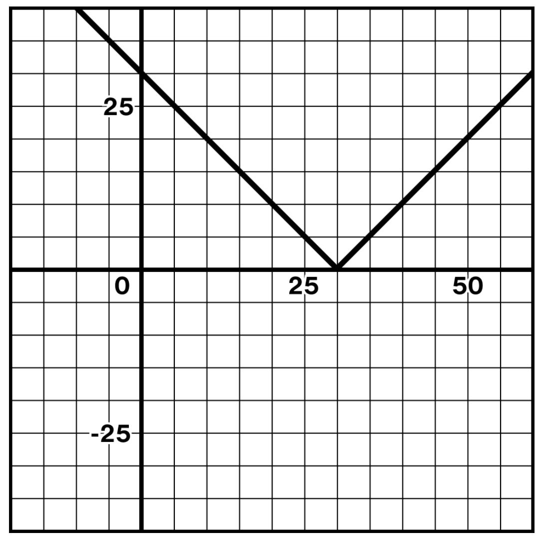
For example, Mr. DeAndre asked his students to guess a mystery number and gave each student a score. In this game, their score was how far away their guess was from the mystery number. The function $f(x) = |x - 30|$ gave the score for each guess, x.

What is the value of $f(25)$? What does it mean?

$$\begin{aligned} f(25) &= |25 - 30| \\ &= |-5| \\ &= 5 \end{aligned}$$

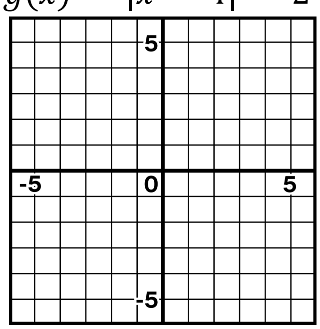
A student who guessed 25 *was* 5 *away from the mystery number.*

What is the value of $f(40)$ and what is its meaning?



Identifying the minimum or making a table can be helpful in making a graph or writing an equation of an absolute value function.

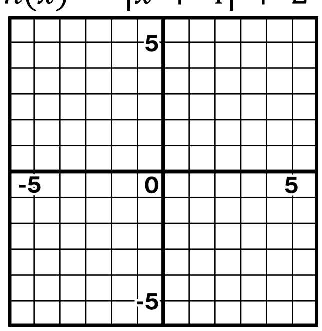
$g(x) = |x - 4| - 2$



x	$g(x)$
4	−2
3	−1
5	−1
0	2

Minimum: $(4, -2)$

$h(x) = |x + 4| + 2$



x	$h(x)$

Minimum:

Things I Want to Remember

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Unit A1.4, Describing Functions: Notes Name ______________________________

Lessons 15–16: Absolute Value Functions

Try This!

$a(x) = |x - 6|$

1.1 What is $a(8)$?

1.2 What is $a(-2)$?

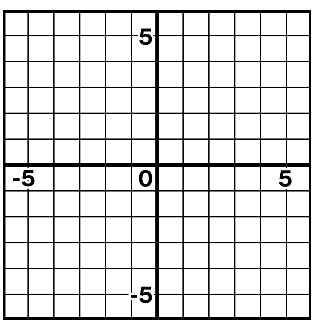
$b(x) = |x + 4| - 1$

2.1 What is $b(6)$?

2.2 What is $b(-4)$?

Complete the missing graphs and minimums. Use the tables if they help with your thinking.

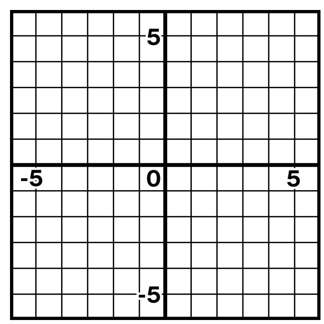
3.1 $c(x) = |x + 3| - 2$



x	$c(x)$

Minimum:

3.2 $d(x) = |x - 2| + 3$



x	$d(x)$

Minimum:

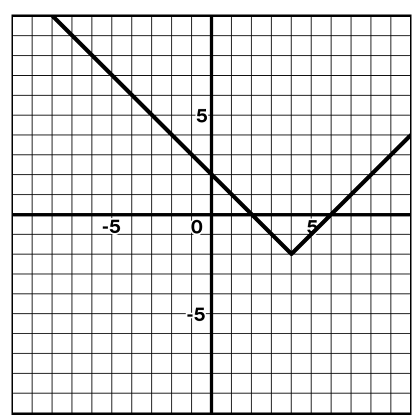
4. Which equation represents this function?

   A. $f(x) = |x| - 2$

   B. $f(x) = |x + 4| - 2$

   C. $f(x) = |x - 2| + 4$

   D. $f(x) = |x - 4| - 2$



- ☐ I can explain how an absolute value function is like the distance from a number.
- ☐ I can calculate and interpret outputs for absolute value functions.
- ☐ I can graph an absolute value function.
- ☐ I can analyze the key features of an absolute value function.

Lesson 17: Using Functions to Tell Stories

Summary

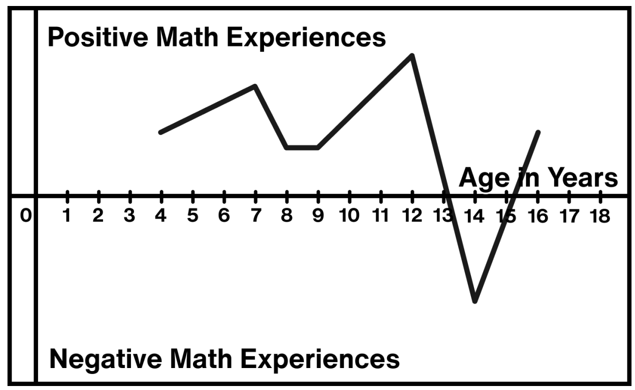
Storytelling is a powerful way to learn more about others and to reflect on one's own journey. Graphing a story can allow for interesting self-discoveries and deeper discussions.

Let's look at Joel's and Isabella's math stories.

Joel is an 11th grade student and a child of military parents. His family is required to move every few years. He graphed his math experiences, $f(x)$, as a function of his age, x.

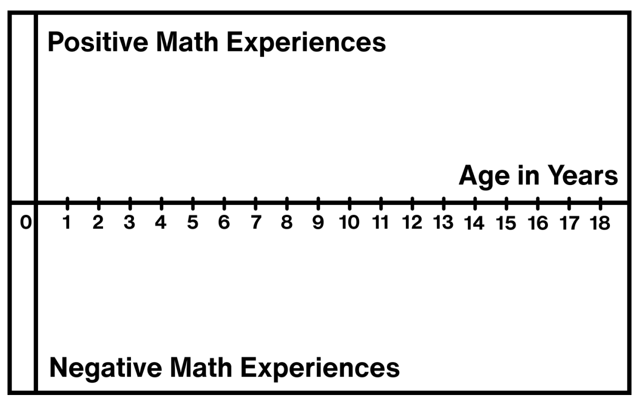
What questions does Joel's graph make you curious to ask?

- *What made your math experiences at age 12 so positive*
-
-



Isabella is a new 9th grade student who recently moved from Puerto Rico. She shared these moments from her math story. Sketch a graph that could represent her story.

- When I was little, I loved finding seashells and organizing them by color and size.
- My sister helped me with my homework and made me feel safe asking for help.
- I moved to the United States when I was 11 and struggled at first to feel at home.
- I was asked to be a peer math tutor by my 9th grade teacher.



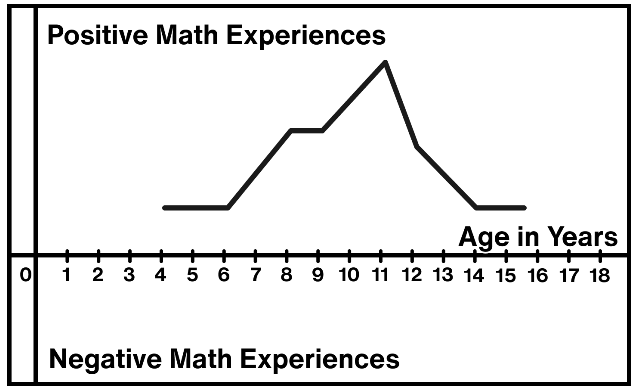
Things I Want to Remember

Lesson 17: Using Functions to Tell Stories

Try This!

Nasir is a 9th grade student. He graphed his math experiences, $f(x)$, as a function of his age, x.

What does each statement say about Nasir's math experiences?



1.1 $f(4) = f(15)$

1.2 $f(x)$ is always positive.

2. Neena is a 10th grade student. Read through her math experience reflection. Then create a graph that could represent her math story.

What is the earliest math experience you remember?

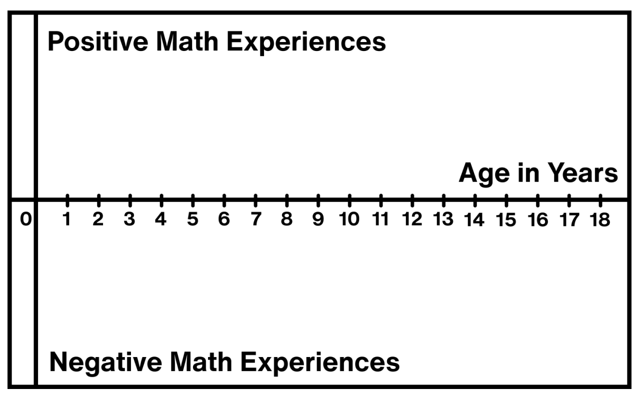
When I was 5, I enjoyed counting how long I could last on my jump rope.

When did your experiences in math class change the most?

In 3rd grade I failed a timed math test and got nervous for every test afterwards.

What was the best experience you had with math?

During middle school, I had fun doing projects that combined science and math.



- ☐ I can use function tools to interpret, describe, and graph real-life relationships.
- ☐ I can use function tools to create graphs of functions to represent real-life relationships.